A passivity approach in port-Hamiltonian form for formation control and velocity tracking

Ningbo Li[1], Jacquelien Scherpen[1], Arjan van der Schaft[1], Zhiyong Sun[2]

1. *Jan C. Willems Center for Systems and Control, University of Groningen*

2. *Department of Electrical Engineering, Eindhoven University of Technology*

1. Background and Problem Formulation

- Preliminaries
- Problem formulation

2. Main results

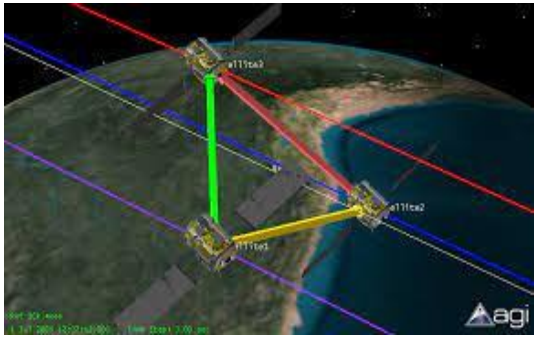
- Passivity-based displacement-based formation
- Passivity-based rigid formation

3. Simulations

4. Conclusions and future research

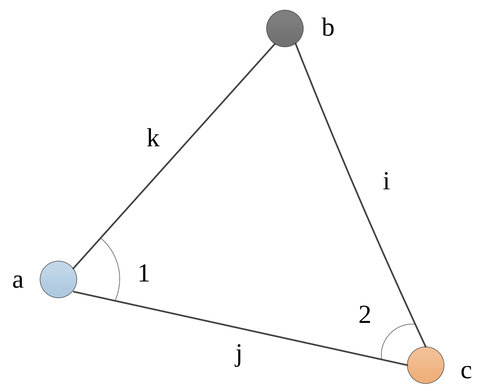
1.1 Preliminaries

Formation control [1-2]: The prescribed group behavior is to achieve a geometrical shape for a network of agents (i.e., to achieve a formation)



[1] *M. Arcak. Passivity as a design tool for group coordination. IEEE Transactions on Automatic Control, 52(8):1380–1390, 2007.*
[2] *Oh, K. K., Park, M. C., & Ahn, H. S. A survey of multi-agent formation control. Automatica, 53, 424-440,2015.*

Formation Classification: Based on the geometric variables that define the shape.



Position: $q_a, q_b, q_c \in \mathbb{R}^d$

Displacement: $z_k = q_a - q_b$

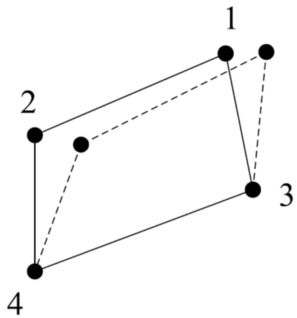
Distance: $||z_k|| = ||q_a - q_b||$

Bearing: $s_k = \frac{z_k}{||z_k||}$

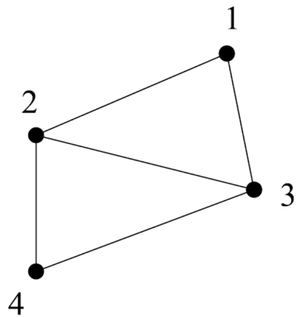
Displacement-based formation: z_k^*, z_i^*, z_j^*

Distance-based formation: $||z_k||^*, ||z_i||^*, ||z_j||^*$

Bearing-based formation: s_k^*, s_i^*, s_j^*



(a) Not rigid.



(b) Rigid.

Input-state-output port-Hamiltonian model [3]:

$$\dot{x} = (J(x) - R(x))\frac{\partial H(x)}{\partial x} + g(x)u$$

$$y = g^T(x)\frac{\partial H(x)}{\partial x}$$

State $x \in \mathbb{R}^n$, input $u \in \mathbb{R}^m$, output $y \in \mathbb{R}^m$, Hamiltonian $H(x)$, interconnection matrix $J(x) = -J^T(x) \in \mathbb{R}^{n\times n}$, dissipation matrix $R(x) = R^T(x) \geq 0 \in \mathbb{R}^{n\times n}$.

Passivity property:

$$\dot{H}(x) \leq y^T u$$

[3] *Van Der Schaft, A. and Jeltsema, D., 2014. Port-Hamiltonian systems theory: An introductory overview. Foundations and Trends in Systems and Control, 1(2-3), pp.173-378.*

1.2 Problem formulation

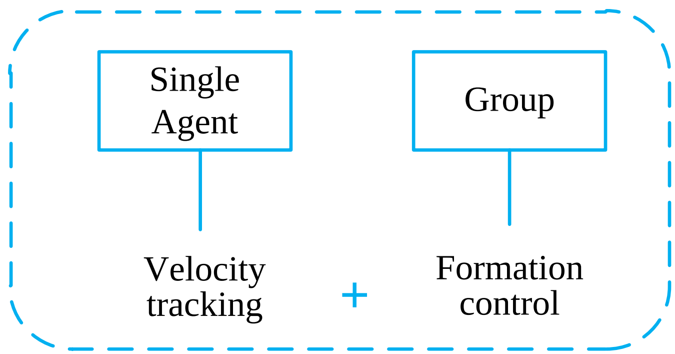
Consider a group of N agents in $\mathbb{R}^d$ with an underlying connected graph $\mathcal{G}(\mathcal{V}_N, \mathcal{E}_M)$. The dynamics of agent i are given by a double integrator model in port-Hamiltonian form.

$$\begin{aligned} \begin{pmatrix} \dot{q}_i \\ \dot{p}_i \end{pmatrix} &= \begin{pmatrix} 0 & I_d \\ -I_d & -D_i^r \end{pmatrix} \begin{pmatrix} \frac{\partial H_i}{\partial q_i} \\ \frac{\partial H_i}{\partial p_i} \end{pmatrix} + \begin{pmatrix} 0 \\ I_d \end{pmatrix} U_i, \\ Y_i &= \tfrac{\partial H_i}{\partial p_i}(p_i), \quad H_i = \tfrac{1}{2} p_i^T M_i^{-1} p_i. \end{aligned}$$

Objective: The whole group achieves a desired shape and each agent tracks a common desired velocity.

Control objectives:

The structure of the controller



1). The velocity of each agent converges to a prescribed common value, i.e.,

$$\lim_{t\to\infty} |\dot{q}_i(t) - v^*| = 0.$$

2). The difference variables associated with the edges

$$z = (B^T \otimes I_d)q, ||z||, \frac{z}{||z||}$$

converge to a prescribed compact set.

1. Background and Problem Formulation

- Preliminaries
- Problem formulation

2. Main results

- **Passivity approach for displacement-based formation**
- **Passivity approach for rigid formation**

3. Simulation

4. Conclusions and future research

2.1 Passivity approach for displacement-based formation

Velocity Tracking

The desired momentum of agent i is given as $p_i^* = M_i v_i^*$. The Hamiltonian of agent i for velocity tracking is difined as

$$H_i^v = \frac{1}{2}(p_i - p_i^*)^T M_i^{-1}(p_i - p_i^*) = \underbrace{\frac{1}{2}p_i^T M_i^{-1} p_i}_{H_i} \underbrace{- p_i^T v_i^* + \frac{1}{2}{v_i^*}^T M_i v_i^*}_{H_i^u}.$$

To eliminate the tracking error, the corresponding control law consisting of two terms is defined as

$$u_i^v = -D_i^r \frac{\partial H_i^u}{\partial p_i} - D_i^t \frac{\partial H_i^v}{\partial p_i} = -D_i^r v^* - D_i^t M_i^{-1} \bar{p}_i.$$

Formation control

The corresponding Hamiltonian is equal to the virtual potential energy of the edges, and is given by

$$H^f = \frac{1}{2}\sum_{j=1}^{M} \bar{z}_j^T \bar{z}_j,$$

where $\bar{z}_j = z_j - z_j^*$.

The dynamics of the formation controller associated with edges are defined by

$$\begin{aligned} \dot{z}_j &= \omega_j, \\ \tau_j &= \frac{\partial H_j^f}{\partial z_j} + D_j^f \omega_j, \end{aligned} \tag{1}$$

The interconnection of the original system and the controllers is established by incidence matrix B, in the compact form given by

$$\begin{aligned} u^f &= -(B \otimes I_d)\tau, \\ \omega &= (B^T \otimes I_d)y. \end{aligned} \tag{2}$$

Combined with (1) and (2), the control law for formation stabilization follows directly as

$$u^f = -(B \otimes I_d)\bar{z} - (B \otimes I_d)D^f(B^T \otimes I_d)M^{-1}p,$$

Theorem 1: Consider a group of agents modeled by double integrators in port-Hamiltonian form, and assume that the graph is undirected and connected. Then the control law $u^v + u^f$ achieves the desired formation while each agent tracks the desired velocity.

Under the proposed framework, the closed-loop system converges to the invriant set

$$\mathcal{E} = \{(z, \xi) | \xi = 0, (B \otimes I_d) \frac{\partial H^f}{\partial \bar{z}} = 0\},$$

where $\xi = \dot{q} - \mathbf{1}_N v^*$.

Acyclic graph: B is full column rank,which implys $\bar{z} = 0$.

Cyclic graph: $\bar{z} \in \ker(B \otimes I_d)$ and $z \in \mathcal{R}(B^T \otimes I_d)\}$ imply $\bar{z} = 0$.

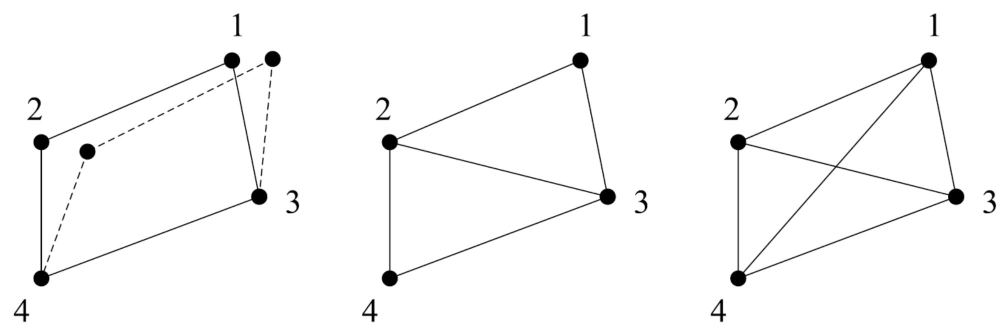
The displacements associated with the edges

$$z = (B^T \otimes I_d)q$$

converge to a prescribed compact set $\Xi \subset \mathbb{R}^{M \times d}$, where $\Xi = \{z_1^*, z_2^*, ..., z_M^*\}$.

2.2 Passivity approach for rigid formation

Distance-based formation



(a) Not rigid. (b) Rigid. (c) Globally rigid.

The distance of edge j that associates agents i and k is defined as

$$||z_j|| = ||q_i - q_k||.$$

In terms of the distance rigidity, the edge function is defined as

$$h_d = [||z_1||^2, ||z_2||^2, ..., ||z_M||^2]^T.$$

The time-derivative of h_d is given as

$$\dot{h}_d = \frac{\partial^T h_d}{\partial q}\dot{q} = \text{blkdiag}(z_1^T, z_2^T, ..., z_M^T)(B^T \otimes I_d)\dot{q},$$

where $R_d = \frac{\partial^T h_d}{\partial q} \in \mathbb{R}^{M\times Nd}$ is defined as the distance rigidity matrix.

Lemma 1[4]: Assume the number of agents N is greater than the dimension d. A framework $f = (\mathcal{G}_N(\mathcal{V}_N, \mathcal{E}_M), q)$ is infinitesimally distance rigid (IDR) in $\mathbb{R}^d$ if and only if $rank(R_d) = dN - d(d+1)/2$.

[4] *Anderson, B.D.O, Yu, C., Fidan, B. and Hendrickx, J.M., 2008. Rigid graph control architectures for autonomous formations. IEEE Control Systems Magazine, 28(6), pp.48-63.*

To make the distance of each edge go to the desired value, the Hamiltonian for formation stabilization is defined as

$$H^d = \sum_{j=1}^{M} H_j^d = \frac{1}{4}\sum_{j=1}^{M}(e_j^d)^2 = \frac{1}{4}\sum_{j=1}^{M}(||z_j||^2 - ||z_j^*||^2)^2.$$

The control law of edge j is proposed as

$$u_j^d = -\frac{\partial H_j^d}{\partial z_j} = -z_j e_j^d.$$

Using the incidence matrix B, the interconnection between the original system and the controller in compact form is defined as

$$u^d = -(B \otimes I_d)\frac{\partial H^d}{\partial z} = -(B \otimes I_d)(\text{blkdiag}(z_1^T, z_2^T, ..., z_M^T))^T e^d = -R_d^T e^d$$

Theorem 2: Consider a group of agents modeled by double integrators in port-Hamiltonian form. If the desired framework $f = (\mathcal{G}_N(\mathcal{V}_N, \mathcal{E}_M), q^*)$ is IDR, then using the control law $u^v + u^d$, the distances of the edges converge to the desired values locally and asymptotically, and each agent tracks the desired velocity.

The distances converge to the invariant set:

$$\{e^d | -(B \otimes I_d)(\text{blkdiag}(z_1^T, z_2^T, ..., z_M^T))^T e^d = -R_d^T e^d = 0\}$$

$$e^d = [e_m^d; e_r^d]$$

Minimally and infinitesimally distance rigid framework: R_d is full row rank, $M = Nd - d(d+1)/2$, R_d^T is full column rank, implying $e_m^d = 0$ on the invariant set.

Infinitesimally distance rigid framework defines the same geometric shape as minimally and infinitesimally distance rigid framework does, so $e_r^d = 0$.

Bearing-based formation

The definition for the bearing of edge j that associates agents i and k is given by

$$s_j = \frac{q_i - q_k}{||q_i - q_k||}.$$

In terms of bearing rigidity, the bearing edge function is defined as

$$h_b = [s_1^T, s_2^T, ..., s_M^T]^T.$$

The time-derivative of h_b is given as

$$\begin{aligned}\dot{h}_b &= \frac{\partial^T h_b}{\partial q}\dot{q} \\ &= \mathrm{blkdiag}(P_{s_1}/||z_1||, P_{s_2}/||z_2||, ..., P_{s_M}/||z_M||)(B^T \otimes I_d)\dot{q},\end{aligned}$$

where $P_{s_1} = I_d - s_1 s_1^T$ is the orthogonal projection matrix which projects vectors onto the orthogonal complement of s_1, and $R_b = \frac{\partial^T h_b}{\partial q}$ is defined as the bearing rigidity matrix.

To make the bearing of each edge go to the desired value, the Hamiltonian for formation stabilization is given as

$$H^b = \sum_{j=1}^{M} H_j^b = \frac{1}{2}\sum_{j=1}^{M}(e_j^b)^T e_j^b = \frac{1}{2}\sum_{j=1}^{M}(s_j - s_j^*)^T(s_j - s_j^*).$$

The output of the controller for edge j is given as

$$u_j^b = -\frac{\partial H_j^b}{\partial z_j} = -(s_j - s_j^*)P_{s_j}/||z_j||.$$

Using the incidence matrix B, the interconnection between the original system and the controller in compact form is defined as

$$\begin{aligned} u^b &= -(B \otimes I_d)\frac{\partial H^b}{\partial z} \\ &= -(B \otimes I_d)\text{blkdiag}(P_{s_1}/||z_1||, \ldots, P_{s_M}/||z_M||)(s - s^*) \end{aligned}$$

Theorem 3: Consider a group of agents modeled by double integrators in port-Hamiltonian form. If the framework $f = (\mathcal{G}_N(\mathcal{V}_N, \mathcal{E}_M), q^*)$ is infinitesimally bearing rigid, then using the control law $u^v + u^b$, the bearings of the edges achieve the asymptotic stability of the desired sets and each agent tracks the desired velocity except for the initial condition where $s(0) = -s^*$.

The bearings converge to the invariant set:

$$\{e^b | (B \otimes I_d) \text{blkdiag}(P_{s_1}/||z_1||, ..., P_{s_M}/||z_M||)(s - s^*)\}$$

Since $P_{s_j} s_j = 0$ and $e_j^b = s_j - s_j^*$, it implies that on the invariant set

$$\{s| - (B \otimes I_d) blkdiag(P_{s_1}, ..., P_{s_M}) s^* = 0\}.$$

Lemma 2 [5]: If the framework $f = (\mathcal{G}_N(\mathcal{V}_N, \mathcal{E}_M), q^*)$ is infinitesimally bearing rigid, then the dynamics $\dot{\delta}(t) = (B \otimes I_d)\text{blkdiag}(P_{s_1}, ..., P_{s_M})s^*$ has two equilibria $\delta_1 = 0$ and $\delta_2 = -2q^* - (\mathbf{1} \otimes ((\mathbf{1} \otimes I_d)^T q(0)/N))$, where δ_1 is asymptotically stable and δ_2 is unstable.

By invoking Lemma 2, there are two elements $s = s^*$ and $s = -s^*$ on this invariant set.

Furthermore, the almost global convergence of $e^b = 0$ except $s(0) = -s^*$ can be obtained since the infinitesimal bearing rigidity is sufficient to global bearing rigidity [5].

[5] *Zhao, Shiyu, and Daniel Zelazo. "Bearing rigidity and almost global bearing-only formation stabilization." IEEE Transactions on Automatic Control 61.5 (2015): 1255-1268.*

1. Background and Problem Formulation

- Preliminaries
- Problem formulation

2. Main results

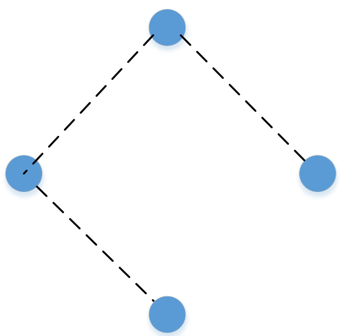
- Passivity approach for displacement-based formation
- Passivity approach for rigid formation

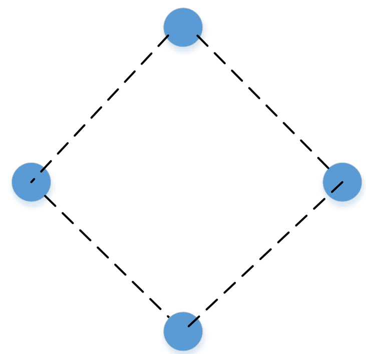
3. Simulation

4. Conclusions and future research

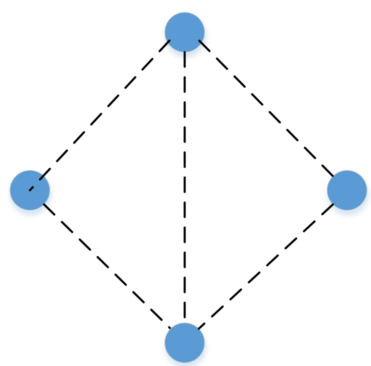
3. Simulation

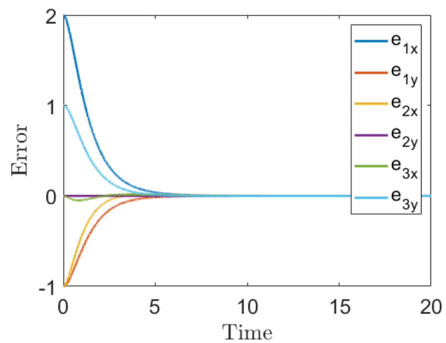
Displacement: Four agents interconnected by a line (acyclic) graph and a (cyclic) ring graph .



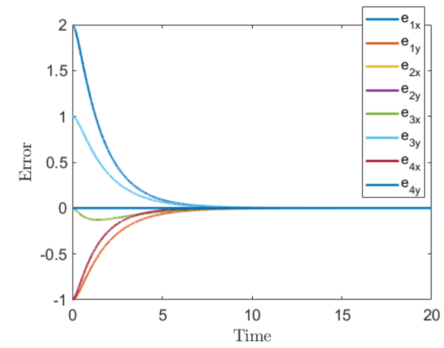


Distance and Bearing: Four agents interconnected by the following graph.

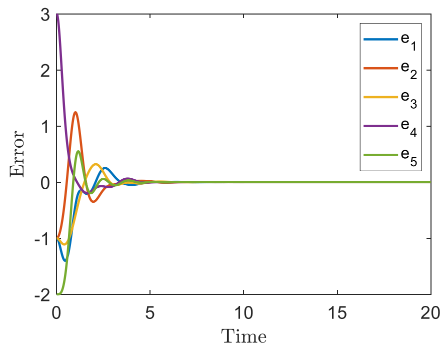




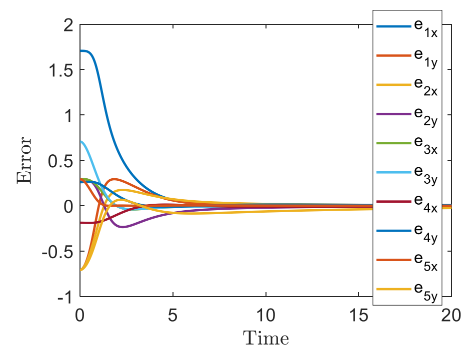
a) Displacement-based formation (acyclic graph)



b) Displacement-based formation (cyclic graph)



c) Distance-based formation



d) Bearing-based formation

Fig. 1 Evolution of formation errors in formation shape stabilization

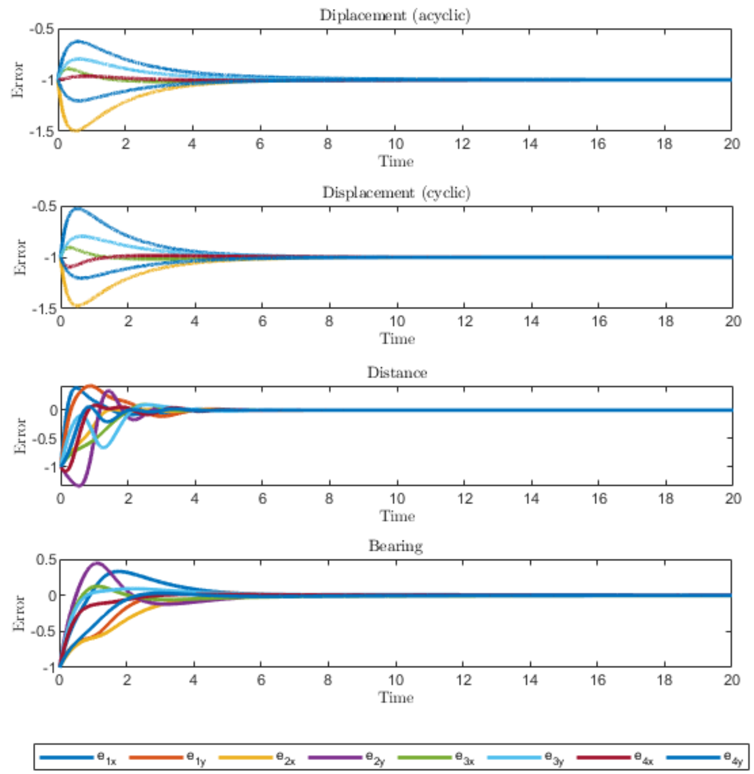


Fig. 2 Evolution of velocity tracking errors

1. Background and Problem Formulation

- Preliminaries
- Problem formulation

2. Main results

- Passivity approach for displacement-based formation
- Passivity approach for rigid formation

3. Simulation

4. Conclusions and future research

4.1 Conclusions

1) Displacement-based formation: Applicable to not only the acyclic graphs but also the cyclic graphs.

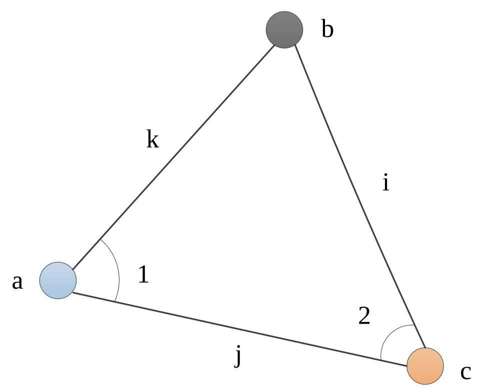
2) Distance-based formation: Local convergence is guaranteed.

3) Bearing-based formation: Almost global convergence is obtained.

4.2 Extensions

1) Angle-based formation.

2) Heterogeneous measurements and dynamics



Thanks for your attention!